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Relevance of the elastic exponent to the thickness of porous plates

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Abstract. We performed an experiment on the modulus exponent of porous plates of Plexiglas, aluminium, steel and brass which covers materials used in most previous measurements by others with similar sample preparation, but different sample thicknesses. Comparisons of the results from this experiment with those of previous work show that the elastic exponent is highly thickness sensitive. Therefore, when measurement of elastic exponent is carried out in order to compare with theoretical predictions, one must make sure that such a thickness effect is negligible.

The elasticity of percolation systems has attracted increasing attention since de Gennes [1] suggested that the elastic modulus of a percolation network with a unique isotropic force constant has the same scaling dependence as the electrical conductivity. Later efforts [2, 3] to study a bond-bending force system, which is believed to dominate the elastic behaviour of general solid in the scaling regime, have implied a new exponent, which is found to be considerably larger than the conductivity exponent. Various 2D experiments [4–7], however, yield diverse values, and most of the results seem, more or less, lower than the theoretical predictions [2, 3, 8]. Chelidze *et al* [7] suggested that the large thickness of samples might be the reason for the discrepancies, but no further argument or concrete comparison is given. In this paper, we report measurements on the elastic exponents of porous plates of different materials which cover most of those used in the previous experiments. The samples are prepared in the way as for experiment by Benguigui *et al* [4], and the thicknesses of the samples are deliberately chosen to be different from those in previous experiments. The experiment itself and comparisons with previous results show that the elastic exponent of porous plates is strongly thickness relevant.

In the present experiments, we measured the bulk modulus E in porous plates of Plexiglas, aluminium, steel and brass under various porosities p near the percolation threshold p_c. E is determined from the slope of the linear part in the load-displacement curve. Under a predetermined probability p, holes are randomly punched on the sites of a 20×20 squre lattice in the middle of the sample. Two different configurations of hole cluster generated by a simple BASIC program are used in preparing samples in order to decrease statistical fluctuation. The sample's size is 150 mm $\times$ 100 mm, and the hole's diameter is slightly larger than 5 mm, the spacing between neighbouring sites. The disposition ensures that the interval between neighbouring sites be cut in case they are

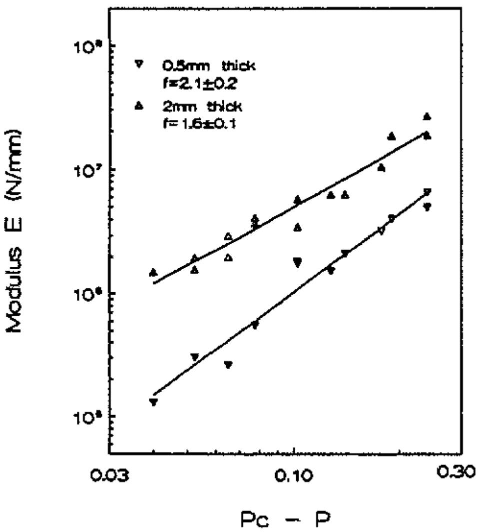

Figure 1. Double-logarithmic plot of the bulk modulus E of brass against $p_c - p$. The full symbols represent the average data of two samples with different configurations of holes.

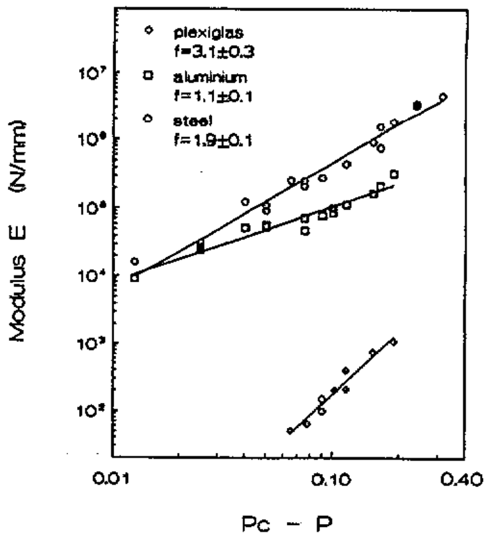

Figure 2. Double-logarithmic plot of the bulk moduli E of Plexiglas, aluminium and steel.

subject to being punched. Samples prepared in this way are believed to be of a percolation configuration with $1 - p$ representing the rigidity. At $p = p_c$, holes coalesce into a spanning cluster while the solid phase becomes two separated segments. Samples with different p are stretched in a testing machine and the corresponding load–displacement curve is obtained through a pressure sensor and a strainmeter. Suppose that $E \sim (p_c - p)^f$ in the scaling regime; then f is determined from the linear regression of the double-logarithmic plot of E against $p_c - p$.

Figure 1 shows the results for brass plates of 2 and 0.5 mm thickness. The exponents are determined, respectively, as 1.6 ± 0.1 and 2.1 ± 0.2, revealing the trend that, the thicker the sample, the larger the exponent. Figure 2 shows results for three other groups of samples. The same situation exists when these results are compared with previous experiments. The modulus exponents are determined as 3.1 ± 0.3 for Plexiglas, 1.1 ± 0.1 for aluminium and 1.9 ± 0.1 for steel. Compared with the accepted theoretical value of 3.67 for bond-lattice models [2, 3] and 5.17 for continuum media [8], the observed values are very low, especially for brass, aluminium and steel. Referring to the previous experiments, we notice that the aluminium samples that Benguigui [4] used is 0.2 mm thick and the corresponding exponent is 3.1 ± 0.5, considerably larger than that of our samples 2 mm thick. For the material Plexiglas, the thickness of our sample is 2 mm, and the measured exponent is 3.1 ± 0.3, while the samples that Chelidze *et al* [7] used are 15 mm thick and the corresponding exponent is 2.13. In addition, the steel samples in the experiment by Lobb and Forrester [6] are 0.13–0.38 mm thick and the corresponding exponent they estimated is 4.95 ± 1.1, while our samples 2 mm thick give an exponent value of 1.9 ± 0.1. The difference between the thicknesses is distinct and, correspondingly, the difference between the exponents appears also to be marked. It is therefore evident that the thickness of the sample is a sensitive factor affecting the measured value of the elastic exponent.

The thickness effect occuring in the measurements of the elastic exponent of porous plate could be qualitatively understood from the following argument. When p approaches p_c, the sample would become more and more ramified, and the number of weaker connections increases, which implies that the area of weaker region will become smaller, and so the surface tension will decrease compared with the bulk strength. Since, the thicker the sample is, the weaker the participation of surface tension in the total resistance against applied stretching would be, therefore, the elasticity of thinner samples would decrease faster than that of thicker samples for p approaching p_c, thereby resulting in a larger value of the exponent.

The extent of the thickness effect could be represented by the ratio w/t, where w is the width of weaker connections in the sample and t is the thickness of the sample; the ratio in part reflects the state of stress in stress concentration region (weaker region) of the samples. The stress state of a stretched 2D system should be appropriately described as plane stress under which $w \gg t$ is required. The weaker connection in our samples is the spacing between the second-nearest-neighbour holes, $w \simeq 4$ mm, and therefore $w/t \simeq 2$, that is far from the state of plane stress. So the fact that the observed exponents in our experiment are considerably lower than the theoretical predictions does not seem to be quite so surprising. In the experiment by Benguigui [4], $w \simeq 2$ mm and $t = 0.2$ mm; thus $w/t \simeq 10$, which more nearly approaches the plane stress state and, correspondingly, the exponent is observed to be nearer to the predicted values than we obtained. Similar phenomena could also be observed in the comparisons of our experiments on Plexiglas and steel with those by Chelidze *et al* [7] and by Lobb and Forrester [6] with w regarded as the average width of the weaker connections. Since the extent of participation of surface tension in the total resistance against stretching is different for different materials, the thickness sensitivity is also related to the characteristics of materials employed. That is why the observed values of exponent differ markedly for different materials with the same thickness. Sieradzki and Li [5] also carried out a measurement of elastic exponent on aluminium. The samples that they used were 2 mm thick, the same as ours, but the value of the exponent that they determined is 3.1 ± 0.2, considerably larger than that obtained from our aluminium samples. This subtle difference may result from the difference between the configurations of the two sets of samples. The samples that they used strictly correspond to a hexagonal bond lattice, while samples in the experiments of others, including ours, are regarded rather as continuum media. Different configurations in the sample should have different thickness sensitivities.

We conclude this experiment by emphasizing that, when measurements of the elastic exponent are carried out to compare with theoretical predictions, one must be sure that the samples are thin enough that the thickness effect could be negligible. Otherwise, the comparison might be inappropriate. In practice, samples of various thicknesses should be included in the experiment in order to find out the specific thickness under which the exponent is thickness insensitive.

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